Closing *Tuesday*: 2.5-6
Closing Friday: 2.7-8, 2.8
Today: 2.6 and 2.7

Note: $\sqrt{x^2} = x, \text{if } x \geq 0,$ and
$\sqrt{x^2} = -x, \text{if } x < 0.$

Entry Task: Find

1. $\lim_{x\to\infty} 3^{(1-x)} + 2^{\left(1+\frac{1}{x}\right)}$

2. $\lim_{x\to\infty} \frac{1 + 7e^{(3x)}}{2e^x + 4e^{(3x)}}$

3. $\lim_{x\to\infty} \frac{2x + \sqrt{1 + x^2}}{5 + 4x}$

4. $\lim_{x\to-\infty} \frac{2x + \sqrt{1 + x^2}}{5 + 4x}$

5. $\lim_{x \to \infty} \left(\sqrt{3 + 5x + 4x^2} - 2x \right)$

Strategies to compute: $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right]$

Special note: If given two fractions, combine them (common denom).

Try plugging in the value:

1. **If denominator ≠ 0, done!**
2. **If denom = 0 & numerator ≠ 0**, the answer is -∞, +∞ or DNE. Examine the sign of the output from each side.
3. **If denom = 0 & numerator = 0**, Use algebra to simplify and cancel until either the numerator or denominator is not zero.

Strategy 1: Factor/Cancel
Strategy 2: Simplify Fractions
Strategy 3: Expand/Simplify
Strategy 4: Multiply by Conjugate
(if you see radicals)

Strategies to compute: $\lim_{x \to \infty} f(x)$

Special note: Combine into one fraction (might need conjugate if given two terms involving a radical).

1. Is it a known limit?

$$\lim_{x \to \infty} \frac{1}{x^a} = 0, \text{if } a > 0; \quad \lim_{x \to \infty} e^{-x} = 0;$$

$$\lim_{x \to \infty} \ln(x) = \infty; \quad \lim_{x \to \infty} \tan^{-1}(x) = \frac{\pi}{2}.$$

2. Rewrite in terms of known limits:

Strategy 1: Multiply top/bottom by $\frac{1}{x^a}$, where *a* is the largest power.
Strategy 2: Multiply top/bottom by e^{-rx}.

Special note:
If *x* is positive, then $x = \sqrt{x^2}$.
If *x* is negative, then $x = -\sqrt{x^2}$.

Note:

After you complete the homework, you can attempt 30 more limit problems directly from old tests using the practice sheet that I compiled on my website.
Here is the direct link:
https://sites.math.washington.edu/~aloveles/Math124Fall2017/m124LimitsPractice.pdf
(you can also see full solutions there)
You can also look at any old midterm or old final for more practice.

2.7: The Derivative at a point

Visual Explanation

For a function, $y = f(x)$, we will define the ***slope of the tangent*** line to $f(x)$ at $x = a$ as follows:

Step 1: Let h represent an arbitrary small number. Draw the *secant* line through the graph at $x = a$ and $x = a + h$.

Step 2: Find the slope of this secant line (your answer will involve *h*).

Step 3: Find the limit as $h \to 0$, we will call this the slope of the tangent line at $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Examples:

1. Find the slope of tangent line to $f(x) = x^2$ at $x = 3$. (Give the equation of the tangent line as well)

2. Find the equation for the tangent line to $g(x) = \sqrt{x-3}$ at $x = 4$.

3. An object is moving on a number line. Assume the position of the object given by $s(t) = \frac{t}{t+1}$ feet.

(a) Find the instantaneous velocity at $t = 3$ seconds.

(b) Find the average velocity from $t = 3$ to $t = 4$ seconds.

Notes:

1. We call $f'(a)$ the **derivative** of $f(x)$ at $x = a$.

2. Graphically, $f'(a)$ is the **slope of the tangent line** to $y = f(x)$ at $x = a$.

3. This is equivalent to saying $f'(a)$ is the **instantaneous rate of change** for $y = f(x)$ at $x = a$.

4. Given $y = f(x)$.
   Units of $f'(a)$ are $\frac{y\text{−}units}{x\text{−}units}$.

For example,
if $x =$ hours and $y = f(x) =$ miles,
then $f'(x) =$ miles/hour.

(Like HW!)

4. Find the equation for the tangent line to the ellipse $x^2 + 4y^2 = 5$ at the point $(x, y) = (1,1)$.